FACULTY OF ENGINEERING
ALEXANDRIA UNIVERSITY

AEJ

(ISSN 1110-0168)

ALEXANDRIA ENGINEERING JOURNAL

PUBLISHED BY : FACULTY OF ENGINEERING
ALEXANDRIA UNIVERSITY - EGYPT

HOSTED BY
ELSEVIER

Available online at www.sciencedirect.com
ScienceDirect

ALEXANDRIA ENGINEERING JOURNAL

Faculty of Engineering, Alexandria University

Actions for selected articles

Select all / Deselect all

Download PDFs

Export citations

Show all article previews

Receive an update when the latest issues in this journal are published

Set up journal alerts

Actions for selected articles
Select all / Deselect all

Download PDFs

Export citations

Show all article previews

Research article • Open access
Numerical prediction of improvement of a Savonius rotor performance with curtaining and fin addition on blade

Mohamed R. Shouman, Mohamed M. Helal, Ahmed A. El-Haroun
Pages 10689-10699

View PDF Article preview

Research article • Open access
Experimental study on performance measurement of planar vacuum membrane dehumidifier with serpentine flow channel designs

Chun-Han Li, Zih-Jyun Lin, Chih-Chang Chang, Saman Rashidi, Wei-Mon Yan
Pages 10701-10711

View PDF Article preview

Research article • Open access
Impact of family presence on healthcare outcomes and patients' wards design

Mohamed Fakhry, Wisam E. Mohammed
Pages 10713-10726

View PDF Article preview

Research article • Open access
Union is Strength: Improving face sketch synthesis by fusing outcomes of Fully-Convolutional-Networks and Random Sampling Locality Constraint

Irfan Azhar, Mudassar Raza, Muhammad Sharif, Seifedine Kadry, Seungmin Rho
Pages 10727-10741

View PDF Article preview

Research article • Open access
Thermal decomposition of a reacting chemical in a stepped rectangular channel with multiple obstacles

Sadia Masud, Nepal Chandra Roy, Litan Kumar Saha
Pages 10743-10755

View PDF Article preview

Research article • Open access
Stability analysis of heat transfer in nanomaterial flow of boundary layer towards a shrinking surface: Hybrid nanofluid versus nanofluid

Aqeel ur Rehman, Zaheer Abbas
Pages 10757-10768

View PDF Article preview

Research article • Open access
Edge detection of ore and rock on the surface of explosion pile based on improved Canny operator

Meiling Huang, Yuelei Liu, Yongmei Yang
Pages 10769-10777

View PDF Article preview

Actions for selected articles
Select all / Deselect all

Download PDFs

Export citations

Show all article previews

Research article • Open access

A study on effectiveness of the variational theory in fluid dynamics applications

Aqsa Riaz, Qazi Mahmood Ul Hassan, Tasawar Abbas, Kaouther Ghachem, ... Lioua Kolsi
Pages 10779-10789

View PDF Article preview

Research article • Open access

A novel wide dual band circularly polarized dielectric resonator antenna for milli meter wave 5G applications

Abinash Gaya, Mohd. Haizal Jamaluddin, Bader Alali, Ayman A. Althuwayb
Pages 10791-10803

View PDF Article preview

Research article • Open access

Start-up and operation of novel EN-MBBR system for sidestreams treatment and sensitivity analysis modeling using GPS-X simulation

Ahmed M. Faris, Haider M. Zwain, Majid Hosseinzadeh, Hasan Sh. Majdi, Seyed Mostafa Siadatmousavi
Pages 10805-10818

View PDF Article preview

Research article • Open access

Warranty and maintenance service contracts for repairable products

B.P. Iskandar, N.F. Sa'idah, U.S. Pasaribu, A. Cakravastia, H. Husniah
Pages 10819-10835

View PDF Article preview

Research article • Open access

A review of supply chain coordination management based on bibliometric data

Jian Xue, Wenjing Zhang, Zeeshan Rasool, Jinhua Zhou
Pages 10837-10850

View PDF Article preview

Research article • Open access

A comparison between fuzzy AHP and fuzzy TOPSIS methods to software requirements selection

Mohd. Nazim, Chaudhary Wali Mohammad, Mohd. Sadiq
Pages 10851-10870

View PDF Article preview

Research article • Open access

Urban physics and outdoor thermal comfort for sustainable street canyons using ANN models for composite climate

Rishika Shah, R.K. Pandit, M.K. Gaur
Pages 10871-10896

View PDF Article preview

Alexandria Engineering Journal (2022) 61, 10819–10835

HOSTED BY

Alexandria University

Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com

Warranty and maintenance service contracts for repairable products

B.P. Iskandar [a],*, N.F. Sa'idah [b], U.S. Pasaribu [c], A. Cakravastia [a], H. Husniah [d],*

[a] *Industrial Engineering, Faculty of Industrial Engineering, Institut Teknologi Bandung, Jalan Ganesha 10, Bandung 40132, Indonesia*
[b] *Doctoral Program in Industrial Engineering and Management, Faculty of Industrial Engineering, Institut Teknologi Bandung, Jalan Ganesha 10, Bandung 40132, Indonesia*
[c] *Statistics Research Division, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Jalan Ganesha 10, Bandung 40132, Indonesia*
[d] *Industrial Engineering, Engineering Faculty, Langlangbuana University, Jalan Karapitan No.116, Bandung 40261, Indonesia*

Received 12 November 2021; revised 8 March 2022; accepted 28 March 2022

KEYWORDS

Maintenance service contract;
Warranty;
Game theory;
Bi-level optimization;
Genetic algorithm

Abstract This paper deals with warranty and maintenance service contracts for repairable products, which involve three parties – i.e., Manufacturer, Service Agent (SA), and Consumer. Six maintenance service contract (MSC) options are proposed and examined, which include an option where the manufacturer sales a package of a product and maintenance services (known as a product service system). The study of MSCs is done from three different perspectives — the Manufacturer, the SA, and the customer. The goal for the study is to determine the optimal price of each option and to select best option (for the consumer), which maximize profits which are a common interest of the three parties involved. The decision problems for the three parties are modeled using a Stackelberg game theory formulation. As the decision problems are interdependent, then the bi-level optimization formulation is used to find best solutions. A genetic algorithm is used to obtain the best solution (including the best price for each option offered and the best option for the consumer). Numerical examples examining six cases are presented to illustrate the best solution.

1. Introduction

Industrial products such machine tools, heavy equipment play an important role to support the main business processes. In a mining company, the heavy equipment (e.g. excavators and dump trucks) are used for loading and hauling mining materials, and the machine tools are the essential part of the production process in a manufacturing company. If the equipment fails while in operation, the business processes will be disrupted, and this will ultimately cause the company to lose either due to a decrease in the production rate or some delays in the delivery of production. To decrease the production disruption due to equipment failures, an effective maintenance is

* Corresponding authors.
E-mail addresses: bermawi@mail.ti.itb.ac.id (B.P. Iskandar), hennie.husniah@gmail.com (H. Husniah).

Peer review under responsibility of Faculty of Engineering, Alexandria University.

https://doi.org/10.1016/j.aej.2022.03.068
1110-0168

Nomenclature

W warranty period
L MSC period
P_1, P_2, P_3, P_4, P_5 price of the contract offered by OEM corresponding to option $i, i = 1, 2, 3, 4, 5$
$P_6, P_7, P_8, P_9, P_{10}$ price of the contract offered by SA corresponding to option $i, i = 1, 2, 3, 4, 5$
T random variable representing time to the first failure
$F(t), f(t)$ distribution function, density function of T
$R(t), r(t)$ cumulative failure intensity function, failure intensity function
β : Shape parameter of Weibull distribution
$r^O(t)\left[r^A(t)\right]$: The initial formula of failure intensity function by OEM[SA]
ψ : Quality of PM action by the SA
$\delta_O[\delta_A]$: Decreasing value of failure intensity function by OEM[SA]
ρ : Multiplier for decreasing-value of failure intensity function by SA
$E[N(T_{i-1}, T_i)]$: Expected failures in (T_{i-1}, T_i)
$C_p^O\left[C_p^A\right]$: PM cost by OEM[SA]
$C_0^O\left[C_0^A\right]$: PM cost (fixed) by OEM[SA]
$C_1^O\left[C_1^A\right]$: PM cost (variable) by OEM[SA]
$C_r^O\left[C_r^A\right]$: CM cost by the OEM[SA]
$D(t)$: Total downtime in $(0, t]$
μ : Parameter of Exponential distribution
$\rho_{M1}, \rho_{M2}, \rho_{M3}, \rho_{M4}, \rho_{M5}$: Expected profit for OEM in option $i, i = 1, 2, 3, 4, 5$
$\rho_{A1}, \rho_{A2}, \rho_{A3}, \rho_{A4}, \rho_{A5}$: Expected profit for SA in option $i, i = 1, 2, 3, 4, 5$
$\Pi_{C1}, \Pi_{C2}, \Pi_{C3}, \Pi_{C4}, \Pi_{C5}$: Expected profit for Consumer in option $i, i = 1, 2, 3, 4, 5, 6$
R, r : Revenue, revenue per time
$\tau_O^*\left[\tau_A^*\right]$: Period to do PM by OEM[SA]
y_1, y_2, y_3, y_4, y_5 : Decision variable for the selected option by OEM (binary), $y_i = 0, 1$
z_1, z_2, z_3, z_4, z_5 : Decision variable for the selected option by SA (binary), $z_i = 0, 1$

needed to ensure that the equipment is always in good condition and has high availability.

Maintenance actions for keeping the good condition of the equipment consist of preventive maintenance (PM) and corrective maintenance (CM) actions. If the company has technological capabilities and resources, these maintenance actions are carried out by the company itself (called in-house maintenance). Otherwise, the maintenance actions are outsourced to an external agent. In addition, many companies prefer to subcontract maintenance services to the external agent due to economic reasons – it is more efficient to outsource. In other words, the company needs to select a maintenance service contract offered by providers (an OEM (original equipment manufacturer) or an agent), which meets its requirement.

Maintenance service contracts (MSCs) have been studied by many researchers and an excellent review can be found in Murthy and Jack [1] in which the MSCs are classified into three categories (i) general, (ii) consumer perspective, and (iii) industry sector. Many different parties (e.g. providers, consumers, underwriters, insurers, service agents and regulators, and governments) are involved in the chain of the MSCs, but the two main parties are providers (sellers of MSCs) and consumers. The providers offer several MSC options and want to determine the price for each option, whilst the consumers (the owner of the equipment) would like to select the best choice when more options are available.

In Murthy and Jack [1] the decision models for the MSCs are grouped into two categories based on the mathematical formulation used to represent the decision problems faced by the two parties involved — (i) game-theoretic (GT) and (ii) non-game-theoretic. Two game theory formulations commonly used are Nash game theory formulation (Laksana and Hartman [2], Lian and Wu [3], Iskandar et al. [4], Husniah et al. [5], and de Santana et al. [6] to name a few) and Stackelberg game theory formulation (Rinsaka and Sandoh [7], and Zhang et al. [8] to name a few).

The GT models can be divided into two categories – namely (i) a single period GT model and (ii) a multi period GT model. In (i), the GT model is static in each period (see, Desai and Padmanabhan [9], Li et al. [10], Kurata and Nam [11], Jiang and Zhang [12], and Heese [13,14]), whilst in (ii) the GT model is dynamic (see, Jack and Murthy [15], Lam and Lam [16], and Hartman and Laksana [17]). Meanwhile, Tarakci et al. [18-20] formulate the decision problems using the non-game-theory formulation. In all three of his papers, Tarakci et al. [18-20] use incentive contract modeling to induce the contractor (consumer) to select the maintenance policy that optimizes the total profit of the manufacturer and contractor.

Most of the papers that study the decision models for the MSCs only involve two parties-providers and customers. In many cases, when an equipment is sold with a warranty, then maintenance services (PM) under warranty can be done, either by an OEM (PM is one package of the warranty) or an external agent (if a warranty only covers a CM action). In the period after the warranty expiry, the owner is fully responsible to maintain the equipment, and both the OEM and the Agent can be involved to provide maintenance services for the equipment. If the consumer does not have resources to carry out maintenance in house, then the consumer needs to select maintenance services offered by the OEM and/or the agent. The works by Gamchi et al. [21] and Esmaili et al. [22] consider the MSC decision models involving three parties (an OEM, an agent, a consumer). However, these two papers deal with the MSCs for non-repairable equipment. For non-repairable equipment, the type of maintenance performed is only corrective replacement (or CM) – as the product is non-repairable. In fact, many durable and industrial products (e.g., cars, machine tools, heavy equipment) are repairable and hence they require maintenance services consisting of PM and CM actions.

This research proposes several MSCs for a repairable product (such as a dump truck) involving three parties - namely the OEM, the service agent (SA), and the consumer (a company requiring the MSC for its product). As a result, this study extends the work by Esmaili et al. [22] to repairable products, and it needs to model (i) the effect of PM action on the product reliability and (ii) the product failure. Moreover, it is considered that a repairable product is sold with warranty, and the warranty service can be provided by either the OEM or the SA as in [22]. After the warranty ends, the consumer can select PM and CM services offered by the OEM and/or the SA. Hence, during the life cycle of the product, the consumer can either select (i) a package of PM and CM services offered by the OEM, (ii) a package of PM and CM services provided by the agent, or (iii) a combination of maintenance services offered by the OEM, and by the agent. As a result, six maintenance service options will be examined, and the price of each option need to be determined such that to maximize profits for the three parties involved. Since there are four options (called a combination option) which involve the OEM and the SA in providing the maintenance services, then the decision problems (to decide the optimal price of those options) for the OEM, and the SA are considered to be interdependent.

Furthermore, this research assumes that the OEM (or manufacturer) is more powerful than the SA, as the OEM has some advantages in the know how for performing maintenance over the SA. And the agent (or the OEM) is more powerful than the customer as there are a few competitors in the market. Then, the decision problems between the OEM (as a leader) and the agent (as a follower), and then between the agent (as a leader) and the consumer (as a follower) can be modelled using a Stackelberg game theory formulation. As a result, two Stackelberg game formulations are built – i.e., (i) OEM Stackelberg game (SG) for the relationship between the OEM and the SA, and (ii) SA Stackelberg game for the relationship between the SA and the consumer, and these two SGs are interdependent. To construct optimization problems for (i) and (ii), a bi-level optimization formulation will be applied as it allows us to integrate the two interdependent decision problems, and this will be described in detail in Section 4. The decision variables in the bi-level optimization problems developed contain several nonlinear functions, and this in turn makes the objective function and constraints of the bi-level optimization become more complex. Therefore, it requires a genetic algorithm (GA) for finding the optimal decisions.

The main contributions resulted from this research are (a) six new MSC options for a repairable product, which extend the maintenance service contract (MSC) options studied by Esmaili, et.al. [22], (b) new decision models involving three parties (the OEM, the SA, the consumer) using a SG formulation, and (c) a new bi-level optimization formulation for finding the optimal decision of MSC options studied and the best MSC option for a repairable product.

This paper is organized as follows: Section 2 provides a problem description and Section 3 presents model formulation including warranty policy, MSC options proposed, failure modeling, PM impact modeling, and the expected profits for each party. In Section 4 describes decision problems, optimization problems, and the GA for seeking the optimal decisions. Section 5 provides numerical examples and discussions of results including managerial insights. Finally, conclusions and a brief description of topics for future research are drawn in the last section.

2. Problem description

A new industrial or durable product such as a car, a dump truck is sold with a warranty. In a common practice, the manufacturer provides a warranty, but now, a third party (agent) also involves in providing a warranty - e.g., BMW, Apple, etc. In this case, the manufacturer offers two options to consumers –i.e., a third-party warranty or a manufacturer warranty (Huang, et al. [33]). In general, warranty only covers a CM (i.e., fixing all failures under warranty with no cost to the consumer) or a PM is excluded in the warranty. Hence, the consumer may outsource the PM to a service agent (SA) or perform it in house. In some cases, the manufacturer offers a PM in one bundle with warranty, or all maintenance (PM and CM) services are done by the manufacturer (or OEM). After the warranty ends, a maintenance responsibility shifts to the consumer. There are two alternatives available for the consumer to maintain the equipment – it is conducted (i) in-house maintenance or (ii) out-sourced to a service agent. In the situation where a company does not have human resources and technology (because it requires a high investment of maintenance facilities) to carry out maintenance, then it is more economical to outsource. Then, the consumer will buy maintenance service to an OEM or a service agent under a maintenance service contract (MSC). For the case where the maintenance services are provided by the OEM, this extends the scope of the OEM's business to include maintenance services not only in the warranty period but also beyond it. As a result, the manufacturer no longer sales only a product but also includes product services (e.g., maintenance and spare part services) required to support the product. In other words, the manufacturer sales a package of a product and maintenance services to consumers (this package is known in literature as a product service system) (Baines, et al. [34]).

This paper proposes six maintenance service options which involve three parties -i.e., the OEM, the SA and the consumer. Two MSC options are a comprehensive coverage of maintenance service (a package of PM and CM service), and four options are a partial maintenance (only PM or CM) or it needs to combine options offered by the OEM and the SA. The study of MSCs will be done from three different perspectives — the OEM, the SA, and the customer. The OEM and the SA needs to decide on the price of each option offered, and the customer needs to select between different options and decide on the best option for its equipment. The goal for the study is to determine the optimal price of each option (for the OEM and/or the agent) and to select best option (for the consumer), which maximize profits which are a common interest of the three parties involved. For the case where the OEM or the agent provides a partial maintenance service over the life cycle of the product, there is some interaction between decision problems of the three parties. Hence, it requires a three stage Game Theory formulation to model an interaction between the manufacturer, agent and customer decisions (this will be discussed in Section 4).

3. Model formulation

This section will present notations, warranty policy, MSC formulation, MSC options considering warranty, failure modeling, and PM impact modeling. After that, the expected profits will be obtained for the three parties involved (the OEM, service agent, and consumer).

3.1. Notations

The following notations will be used to formulate mathematical models needed to study the proposed maintenance service contract:

3.2. Warranty policy

Consider a new durable product (e.g., cars, dump trucks) which is sold with warranty. The warranty period is W(in years) – e.g., $W = 2$ years. The product warranty can be either provided by the manufacturer or by a third party (agent) - e.g., BMW, Apple, etc. Here, the manufacturer offers two warranty options to consumers –i.e., a third-party warranty or a manufacturer warranty (Huang, et al. [33]). In general, warranty only covers a CM (i.e., fixing all failures under warranty) or a PM is not included in the warranty. It is considered that all product failures under warranty are repaired at no cost to the consumer (Free Repair Warranty).

3.3. Maintenance service contract (MSC)

After the warranty expires, the maintenance responsibility shifts to the consumer. To fulfill the maintenance responsibility, the consumer will subcontract maintenance services (PM and/or CM) to the OEM or a service agent under a maintenance service contract (MSC). Hence, there are three parties involved (i.e. manufacturer (or OEM), SA, and consumer) in providing warranty as well as and maintenance services over a lifetime of a product.

Let L be the MSC period. The period is divided into two periods i.e.,– (i) $(0, W)$ and (ii)(W, L). This research considers that the OEM or the SA can provide a comprehensive coverage of maintenance (PM and CM) or a partial maintenance (only PM or CM) in$(0, W)$, and(W, L). As a result, each party (OEM or SA) can offer five options described as follows:

3.4. The OEM options

Option M_1: The OEM provides PM and CM during $(0, L)$ in one bundle at priceP_1.

Option M_2: The OEM provides PM during $(0, L)$ and CM only during $(0, W)$ at priceP_2.

Option M_3: The OEM provides PM and CM during $(0, W)$ in one bundle at priceP_3.

Option M_4: The OEM provides PM only during $(0, W)$ at priceP_4.

Option M_5: The OEM provides CM only during (W, L) at priceP_5.

3.5. The SA options

Option A_1: The SA performs CM only during (W, L) at priceP_6.

Option A_2: The SA performs PM and CM during (W, L) at priceP_7.

Option A_3: The SA performs PM during (W, L) and CM during $(0, L)$ at priceP_8.

Option A_4: The SA performs PM and CM during $(0, L)$ in one bundle at priceP_9.

Option A_5: The SA performs PM during $(0, L)$ and CM only during $(0, W)$ at priceP_{10}.

From the customer's viewpoint, six MSC options are available to be chosen, where four options are combination options offered by the OEM, and the SA (i.e., Options 2, 3, 4 and 6) and two MSC options are a pure MSC option which is a package of PM and CM service offered by the OEM (i.e., Option 1) or the SA (i.e., Option 1) (See Table 1).

The descriptions of the six options are as follows:

Option 1: The OEM offers PM and CM in $(0, L)$ in one bundle at priceP_1.

Option 2: The OEM offers PM in $(0, L)$ and CM in $(0, W)$ at priceP_2, whilst the SA also offers CM only during (W, L) at priceP_6.

Option 3: The OEM offers PM and CM in $(0, W)$ at priceP_3, whilst the SA offers PM and CM in (W, L) at priceP_7.

Option 4: The OEM offers PM in $(0, W)$ at price P_4 whilst the SA offers PM.

in(W, L), and CM in $(0, L)$ at priceP_8.

Option 5: The SA offers PM and CM in $(0, L)$ at priceP_9.

Option 6: The OEM offers CM in (W, L) at priceP_5, whilst the SA offers PM in $(0, L)$ and CM in $(0, W)$ at priceP_{10}.

3.6. Failure modeling

The product considered is an industrial or durable product such as a car, a machine tool, a dump truck. One can model the failure of the product using a black-box approach (observe only functioning or failed state). Suppose that T is a random variable representing time to the first failure, the distribution function of T are $F(t)$ and $f(t)$, and the failure rate and cumulative functions associated with $F(t)$ are $r(t) = f(t)/(1 - F(t))$ and$R(t) = \int_0^t r(x)dx$.

The process of the subsequence failures is influenced by the PM conducted and a type of repair used to restore a failed product, and these will be described in the next section.

3.7. PM impact modeling

A PM is performed periodically at the time $T_i, i = 1, 2, ..., k$ where $T_k = L$. Hence, the total number of PM is k times in $(0, L)$. It is assumed that PM is an imperfect, and any failure occurring between PMs will be fixed with minimal repair (Barlow and Hunter [23]). The effect of an imperfect PM action to improve the reliability of the equipment is described as follows. There are two approaches to modeling the effect of a PM – i.e., through a reduction in (a) virtual age or (b) failure intensity function (Jiang and Murthy [24]). Here, each imperfect PM results in a reduction of failure intensity function.

It is considered that a PM performed by the OEM and SA has different quality, and this needs to model the effect of a PM for each party. Let $r^O(t)$ be the failure rate function of the equipment in $(0, T_1)$ if PM and CM are carried out by the OEM, where$r^O(t) = r(t), 0 < t < T_1$. After the i^{th} PM, the failure rate function becomes$r_i^O(t) = r^O(t) - i\delta_O$, where$0 \leqslant \delta_O \leqslant r_i^O(T_i) - r_{i-1}^O(T_{i-1})$.

Table 1 Possible combination of the OEM, and the SA options forming six options to be chosen by the consumer.

Consumer's option choices (C_i)	Option offers by OEM (M_i)	Option offers by SA (A_i)	(0, W)		(W, L)	
			PM	CM	PM	CM
1	1	–	M	M	M	M
2	2	1	M	M	M	A
3	3	2	M	M	A	A
4	4	3	M	A	A	A
5	–	4	A	A	A	A
6	5	5	A	A	A	M

Furthermore, it is assumed that a PM action carried out by the SA is less effective than that done by the OEM. The effect of a PM done by the SA is modeled as follows. After a PM is done atT_1, the failure rate function is given by$r^A(t) = \psi r^o(t), \psi \geqslant 1, t > T_1$, which increases much faster than$r^O(t)$. The parameter ψ represents the quality of PM action where $\psi > 1$ means that the PM action is not better than a PM done by the OEM ($\psi = 1$) (see Fig. 1). Hence, the failure rate function after the ith PM becomes$r_i^A(t) = r^A(t) - i\delta_A$, where $\delta_A = \rho\delta_O, 0 < \rho \leqslant 1$ (means that the decreasing value of failure rate function is not as better as the PM done by the OEM). If $\delta_O = \delta_A$ or$\rho = 1$, then both the OEM and SA have the same reduction in the failure rate level. Visualization of modeling the PM actions of OEM and SA can be seen in Fig. 1 (a) and (b).

The expected failures during $[T_{i-1}, T_i)$ are given by:

$$E[N(T_{i-1}, T_i)] = \int_{T_{i-1}}^{T_i} r_{i-1}(x)dx = [R(T_i) - R(T_{i-1})] - (T_i - T_{i-1})i\delta \quad (1)$$

3.8. Expected profit

The expected profit over the contract interval for three parties involves (i.e., OEM, SA, and consumer) are obtained as follows.

3.9. Expected profit

The expected profit = the total income earned – the total cost (consisting PM cost, CM cost, and penalty cost). First, these three costs are derived, and then the total income. The PM cost, CM cost, and the expected cost of the penalty will be formulated in advance from Iskandar and Husniah [25].

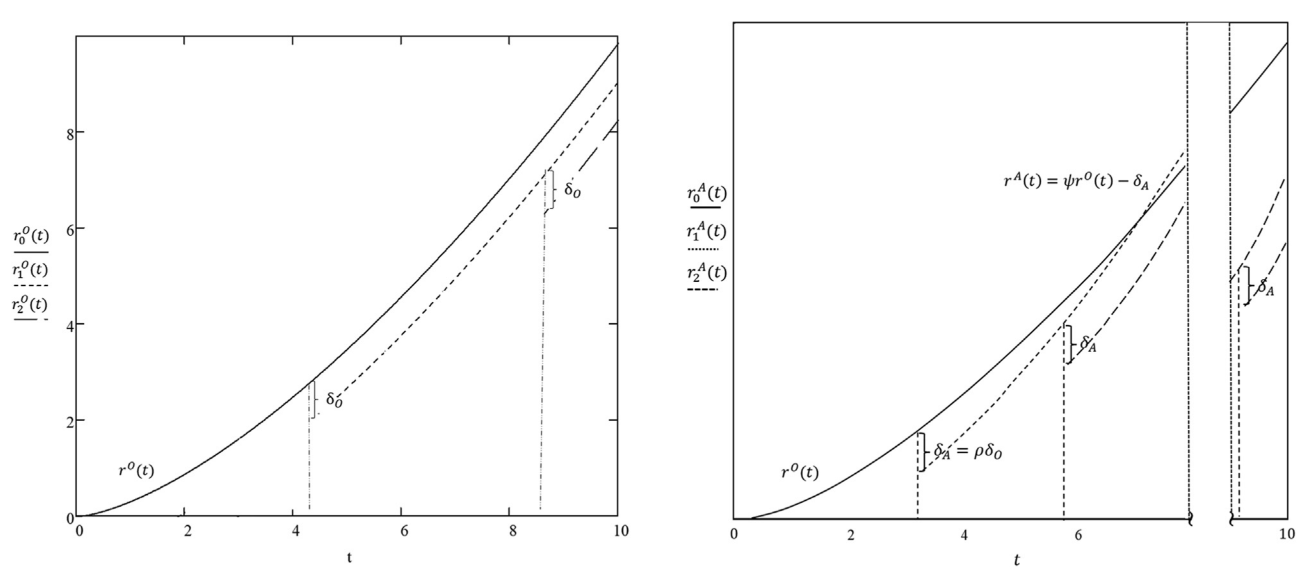
PM Cost: the expected PM costs for OEM and SA are formulated as follows.

The expected PM costs for OEM: $C_p^O(\delta_O) = C_0^O + C_1^O\delta_0$

The expected PM costs for SA: $C_p^A(\delta_A) = C_0^A + C_1^A\delta_A$

Where $C_0^O\left[C_0^A\right]$ and $C_1^O\left[C_1^A\right]$ are a fixed cost and a variable cost if a PM done by the OEM[the Agent], respectively.

Then, the expected total cost of PM if all PMs done by the OEM during $(0, L)$ is given by



(a) (b)

Fig. 1 Imperfect PM with reductions in failure rate function. Every time (a) OEM or (b) SA conducts PM, the failure rate function drops by δ_O andδ_A, respectively. This figure refers to reduction in intensity function by Jiang and Murthy [24].

$$E[N_{PM}] = \sum_{i=1}^{k} C_p^O(\delta_O) = kC_0^O + kC_1^O(\delta_O) \tag{2}$$

The expected total cost of PM if all PMs done by the agent is given in (2) replacing $C_p^O(\delta_O)$ with$C_p^A(\delta_A)$. The total PM actions during $(0, L)$ is$k = m + n$, i.e. m PM during $(0, W)$ and n PM during(W, L). Thus, for PMs done during $(0, W)$ replace k in (2) by m. Whereas, the i in (2) starts from $m + 1$ during(W, L).

CM cost: Suppose that $C_r^O\left[C_r^A\right]$ is the repair cost done by OEM[SA] for each CM. Then, the expected total cost of CM if all CMs done by the OEM during $(0, L)$ is given by.

$$E[N_{CM}] = C_r^O\left(\sum_{i=1}^{k}\int_{T_{i-1}}^{T_i} r_{i-1}^O(t)dt\right) = C_r^O\left(R(L) + \delta_O\left(\sum_{i=1}^{k} T_i - kL\right)\right) \tag{3}$$

The expected total cost of CM if all CMs done by the agent is given in (3) replacing C_r^O, r_{i-1}^O, and δ_O withC_r^A, r_{i-1}^A, and δ_A. For CMs done during $(0, W)$ replace k in (3) by m. Meanwhile, the i starts from $m + 1$ during(W, L).

Downtime formulation: Suppose D(t) as downtime with $D(t) = \sum_{i=1}^{N(t)} X_i$. Random variable X_i states downtime caused by i^{th} failure and N(t) states expected failures along (0, t). Assume that X_i *i.i.d* is with certain distribution function $F_d(t)$. In this paper, downtime is assumed by Exponential distribution with parameter$1/\mu$..

$$E[D(t)] = E\left[\sum_{N(t)}^{i=1} X_i\right] = \sum_{N(t)}^{i=1} E[X_i] = N(t)\cdot(\mu) = \sum_{N(t)}^{i=1}\int_{t,i}^{t,i-1} r_{i-1}(x)dx\cdot(\mu) \tag{4}$$

This expected total downtime needs to be adjusted according to the failure rate function associated with the option under consideration (i.e., whether OEM and/or SA involves in providing the MSC option).

For each of the six options proposed, it needs to obtain the expected profit for the OEM, and the SA.

Option 1: [The OEM offers PM and CM during $(0, L)$ in one bundle with warranty at price P_1].

In this option, maintenance during and after warranty is done by having a contract with the OEM (PM and CM) for P_1. By using the formulation above, the expected profit formulation of each party is as follows:

3.10. Expected profit for OEM

The expected profit for OEM = warranty price (P_1) – [PM cost ($C_p^O(\delta_O)$) × total number of PM actions] – [CM cost (C_r^O) × the expected of total failures in (0, L)].

$$\rho_{M1}(P_1) = P_1 - E[N_{PM}] - E[N_{CM}], \tag{5}$$

where, $E[N_{PM}]$, and $E[N_{CM}]$ is given in (2) and (3).

Expected profit for SA: not available as the SA does not involve to provide maintenance services in$(0, L)$.

3.11. Expected profit for consumer

From a consumer's perspective, the expected profit is derived from the difference in total income (R) obtained by consumers during the contract period with the maintenance contract price chosen by the consumer from the OEM and/or SA. The total income of consumers is obtained from the results of maintenance contracts. For example, in coal companies, revenue is generated from loading and hauling activities. The consumer will increase his income r (per time).

The expected profit for consumers is:

Expected profit for consumers = (revenue (R) — MSC price offered by the OEM (P_1).

$$\Pi_{C1} = R - P_1; R = (r)(L - E[D(t)]) \tag{6}$$

where r is the revenue earned from the operating the equipment and $E[D(t)]$ is given in (4).

Option 2: [The OEM offers (i) PM in $(0, L)$ and (ii) CM in$(0, W)$, whilst the SA offers CM only in (W, L)].

In this option, the OEM offers PM in one package with warranty at priceP_2, which also covers CM during the warranty. The SA also offers CM after warranty. Hence, the OEM's expected profit is:

3.12. Expected profit for OEM

Expected profit for OEM = PM one bundle with warranty (P_2) - (PM cost ($C_p^O(\delta_O)$) × total number of PM actions) - (CM cost (C_r^O) × the expected number of failures in (0, W).

$$\rho_{M2}(P_2) = P_2 - E[N_{PM}] - C_r^O\left(\sum_{i=1}^{m}\int_{T_{i-1}}^{T_i} r_{i-1}^O(t)dt\right), \tag{7}$$

where, $E[N_{PM}]$ is given in (2) and the CM cost formula is given in (3) replacing k by$m$..

Expected profit for SA: The SA carries out CM in(W, L).

Expected profit for SA = Price of CM in (W, L) (P_6) - (CM cost (C_r^A) × the expected number of failures in (W, L).

$$\rho_{A1}(P_6) = P_6 - C_r^A\left(\sum_{i=m+1}^{k}\int_{T_{i-1}}^{T_i} r_{i-1}^O(t)dt\right) \tag{8}$$

Note: the failure rate function after the expiry of warranty follows the one of OEM, because all the PM actions are carried out by the OEM. The CM cost formula is given in (3) with the$i$ starts from $m + 1$.

3.13. Expected profit for Consumer

Expected profit for consumer = (revenue (R) — MSC price of OEM (P_2) – MSC price of agent (P_6).

$$\Pi_{C2} = R - P_2 - P_6 \tag{9}$$

Option 3: [The OEM offers PM in one bundle with warranty in $(0, W)$ at priceP_3, whilst the SA offers PM and CM in (W, L) at price P_7].

3.14. Expected profit for OEM

Expected profit for OEM = MSC price (P_3) - (PM cost ($C_p^O(\delta_O)$) × total number of PM actions) - (cost CM (C_r^O) × the expected number of failures in (0, W):

$$\rho_{M3}(P_3) = P_3 - \sum_{i=1}^{m} C_p^O(\delta_O) - C_r^O\left(\sum_{i=1}^{m}\int_{T_{i-1}}^{T_i} r_{i-1}^O(t)dt\right), \quad (10)$$

where, the PM and CM cost formula is given in (2) and (3) replacing k with$m$..

3.15. Expected profit for SA

Expected profit for SA = MSC price (P_7) - (PM cost ($C_p^A(\delta_A)$) × total number of PM actions) - (CM cost (C_r^A) × the expected number of failures in (W, L).

$$\rho_{A2}(P_7) = P_7 - \sum_{i=m+1}^{k} C_p^A(\delta_A) - C_r^A\left(\sum_{i=m+1}^{k}\int_{T_{i-1}}^{T_i} r_{i-1}^A(t)dt\right), \quad (11)$$

where, the PM and CM cost formula is given in (2) and (3) with the$i$ starts from $m+1$.

3.16. Expected profit for Consumer

Thus, the expected profit for consumers is:

Expected profit for consumer = (revenue (R) — OEM's MSC price (P_3) – Agent's MSC price (P_7).

$$\Pi_{C3} = R - P_3 - P_7 \quad (12)$$

Option 4: [The OEM offers PM only during $(0, W)$ at priceP_4, whilst the SA offers PM in (W, L) and CM in $(0, L)$ at price P_8].

3.17. Expected profit for OEM

Expected profit for OEM = MSC price (P_4)- (PM cost ($C_p^O(\delta_O)$) × total number of PM actions).

$$\rho_{M4}(P_4) = P_4 - \sum_{i=1}^{m} C_p^O(\delta_O), \quad (13)$$

where, the PM cost formula is given in (2) replacing k with$m$..

3.18. Expected profit for SA

Expected profit for SA = MSC price (P_8) - (cost PM ($C_p^A(\delta_A)$) × total number of PM actions) - (CM cost (C_r^A) × the expected number of failures in (0, L).

$$\rho_{A3}(P_8) = P_8 - \sum_{i=m+1}^{k} C_p^A(\delta_A) - E[N_{CM}], \quad (14)$$

where, the PM cost formula is given in (2) with the i starts from $m+1$ and $E[N_{CM}]$ is given in (3).

3.19. Expected profit for consumer

Thus, the expected profit for consumers is:

Expected profit for consumer = (revenue (R) — OEM's MSC price (P_4) – Agent's MSC price (P_8).

$$\Pi_{C4} = R - P_4 - P_8 \quad (15)$$

Option 5: [The SA offers PM and CM in $(0, L)$ at price P_9].

In this option, maintenance during and after warranty is done by having a contract with the Agent (PM and CM) forP_9..

Expected profit for OEM: not available as no maintenance services from the OEM in$(0, L)$.

3.20. Expected profit for SA

Expected profit for SA = warranty price (P_9) - (PM cost ($C_p^A(\delta_A)$) × total number of PM actions) - (CM cost (C_r^A) × the expected number of failures in (0, L).

$$\rho_{A4}(P_9) = P_9 - E[N_{PM}] - E[N_{CM}], \quad (16)$$

where, $E[N_{PM}]$,and $E[N_{CM}]$ is given in (2) and (3).

3.21. Expected profit for consumer

Thus, the expected profit for consumers is:

Expected profit for consumer = (revenue (R) — Agent's MSC price (P_9).

$$\Pi_{C5} = R - P_9 \quad (17)$$

Option 6: [The OEM offers CM only during (W, L) at priceP_5, whilst the SA offers PM in $(0, L)$ and CM in $(0, W)$ at price P_{10}].

3.22. Expected profit for OEM

Expected profit for OEM = warranty price (P_5) - (CM cost (C_r^O) × the expected number of failures in.

(W, L).

$$\rho_{M5}(P_5) = P_5 - C_r^O\left(\sum_{i=m+1}^{k}\int_{T_{i-1}}^{T_i} r_{i-1}^A(t)dt\right) \quad (18)$$

Note: the failure rate function after the expiry of warranty follows the one of SA, because all the PM actions are carried out by the SA. The CM cost formula is given in (3) with the$i$ starts from $m+1$.

3.23. Expected profit for SA

Expected profit for SA = A package of PM and CM at price (P_{10}) - (PM cost ($C_p^A(\delta_A)$) × total number of PM actions) - (CM cost (C_r^A) × the expected number of failures in$(0, W)$.

$$\rho_{A5}(P_{10}) = P_{10} - E[N_{PM}] - C_r^A\left(\sum_{i=1}^{m}\int_{T_{i-1}}^{T_i} r_{i-1}^A(t)dt\right), \quad (19)$$

where, $E[N_{PM}]$ is given in (2) and the CM cost formula is given in (3) replacing$k$ by m..

3.24. Expected profit for consumer

The expected profit for consumers is as follows.

Expected profit = (revenue (R) — OEM's MSC price (P_{10}) – Agent's MSC price (P_5).

$$\Pi_{C6} = R - P_{10} - P_5 \tag{20}$$

4. Optimal decisions

As described in Section 3.3, six MSC options are proposed, of which four options (Options 2, 3, 4, and 6) involve three parties, and two options (Options 1 and 5) two parties. Thus, there are two decision problems – (a) the three-partite decision problem related to options 2, 3, 4, and 6 and (b) the two-partite decision problem related to options 1 and 5. These decision problems will be described as follows.

4.1. Three-partite decision problem

The decision problem situation under consideration is described as follows. The OEM has a dominant position than the SA, as the OEM has some advantages in the know how for performing maintenance over the SA, and in turn the SA (or the OEM) is more powerful than the customer as there are a few competitors of maintenance services in the market (Allain [35]). An example for this situation is that a mining company in a remote area operates dump trucks for hauling mining materials and the maintenance for the trucks is outsourced to the OEM. In providing the maintenance service for the trucks, the OEM then an external agent (e.g., a local company) can be involved and this in general will save cost of the service delivery. Hence, one can model the interaction between the OEM (as a leader) and the SA (as a follower), and then between the SA (as a leader) and the consumer (as a follower) using Stackelberg game theory formulation (see Basar and Olsder [44]). Stackelberg game (SG) is a non-cooperative game in which the leader company decides first and then the follower company reacts to the leader's decision. This SG is appropriate to model decision problems in which the two parties decide the optimal prices in a leader–follower relationship.

As a result, we have (i) OEM Stackelberg game (for the OEM and the SA relationship) and (ii) SA Stackelberg game (for the SA and the consumer relationship), and (i) and (ii) form a three-stage Stackelberg game (see Fig. 2).

As the decisions in (i) and (ii) are interdependent, then it cannot be solved separately. Hence, we use a bi-level optimization formulation to model the interaction between the optimization problems in (i) and (ii), in which the lower-level problem (SA Stackelberg game) is nested in the upper-level problem (OEM Stackelberg game) (Bard [41]; Dempe and Dutta [42]). This will be discussed in detail in Section 4.1.3.

Before proceeding with modeling decision problems for the three parties, it needs to find the optimal PM policy performed by the OEM or SA as the resulting optimal PM solutions will be used as inputs to the bi-level optimization problem.

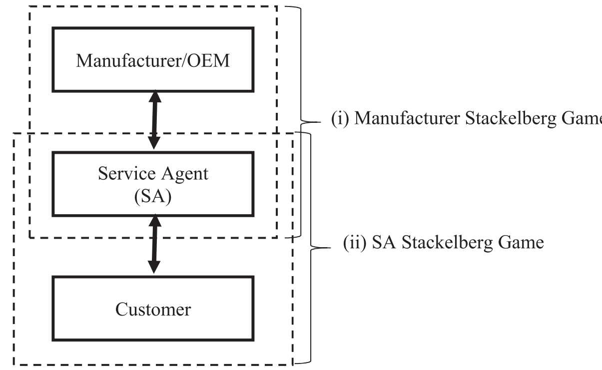


Fig. 2 Three-stage Stackelberg Game Theory Formulation.

4.1.1. Optimal PM policy

In the six MSC options studied, several of which include a PM service done by OEM or SA. Hence, it needs to find the optimal PM policy offered by each party. As mentioned in Subsection 3.5 that a PM is done periodically at $T_i, i = 1, 2, ..., k$ where $T_k = L$. It is assumed that the interval between PMs is constant given by τ, then$T_i - T_{i-1} = \tau, i = 1, 2, ..., k$. Each PM is considered to reduce the failure rate function byδ. Hence, the PM policy can be characterized by two parameters(τ, δ). These two parameters become $(\tau = \tau_O, \delta = \delta_O)$ when the PM is conducted by the OEM, and $(\tau = \tau_A, \delta = \delta_A)$ when the PM is done by the SA.

4.1.1.1. Oem's Optimal PM policy. The optimal (τ_O^*, δ_O^*) is determined to minimize the expected total cost (i.e., PM and CM Costs) given by.

$$TC_O(T_i = \tau_O, \delta_O) = C_p^O(k) + C_r^O\left(R(L) + \delta_O\left(\sum_{i=0}^{k} T_i - kL\right)\right)$$

4.1.1.2. SA's optimal PM policy. The PM policy is characterized by two parameter – i.e.,(τ_A, δ_A), The optimal (τ_A^*, δ_A^*) is determined to minimize the expected total cost (i.e., PM and CM Costs) given by.

$$TC_A(T_i = \tau_A, \delta_A) = C_p^A(k) + C_r^A\left(R_A(L) + \delta_A\left(\sum_{i=0}^{k} T_i - kL\right)\right)$$

4.1.2. Optimization

This research uses a bi-level optimization function in which an optimization function (called the upper-level problem) contains another optimization function (called the lower-level problem) as a constraint (Sinha et al. [43]). In this study, the upper-level optimization function is OEM Stackelberg game representing the interaction between OEM and the SA, and the lower-level optimization is SA Stackelberg game representing the interaction between SA and the consumer, as in Gamchi et al. [21] and Esmaili et al. [22]. The bi-level optimization function is described as follows.

Upper-level problem: OEM-Stackelberg model –.

$$\max \rho_M(P_2,P_3,P_4,P_5) = y_1\left\{P_2 - E[N_{PM}] - C_r^O\left(\sum_{i=1}^{m}\int_{T_{i-1}}^{T_i} r_{i-1}^O(t)dt\right)\right\} + y_2\left\{P_3 - \sum_{i=1}^{m} C_p^O(\delta_O) - C_r^O\left(\sum_{i=1}^{m}\int_{T_{i-1}}^{T_i} r_{i-1}^O(t)dt\right)\right\} + y_3\left\{P_4 - \sum_{i=1}^{m} C_p^O(\delta_O)\right\} + y_4\left\{P_5 - C_r^O\left(\sum_{i=m+1}^{k}\int_{T_{i-1}}^{T_i} r_{i-1}^A(t)dt\right)\right\} \quad (21)$$

s.t.

$$P_6^* = y_1(R - P_2 - \Pi_{C2}) \quad (22)$$

$$P_7^* = y_2(R - P_3 - \Pi_{C3}) \quad (23)$$

$$P_8^* = y_3(R - P_4 - \Pi_{C4}) \quad (24)$$

$$P_{10}^* = y_4(R - P_5 - \Pi_{C5}) \quad (25)$$

$$y_1 + y_2 + y_3 + y_4 = 1 \quad (26)$$

Lower-level problem: SA-Stackelberg model.

$$\max \rho_A(P_6,P_7,P_8,P_{10}) = z_1\left\{P_6 - C_r^A\left(\sum_{i=m+1}^{k}\int_{T_{i-1}}^{T_i} r_{i-1}^O(t)dt\right)\right\} + z_2\left\{P_7 - \sum_{i=m+1}^{k} C_p^A(\delta_A) - C_r^A\left(\sum_{i=m+1}^{k}\int_{T_{i-1}}^{T_i} r_{i-1}^A(t)dt\right)\right\} \quad (27)$$

$$+z_3\left\{P_8 - \sum_{i=m+1}^{k} C_p^A(\delta_A) - E[N_{CM}]\right\} + z_4\left\{P_{10} - E[N_{PM}] - C_r^A\left(\sum_{i=1}^{m}\int_{T_{i-1}}^{T_i} r_{i-1}^A(t)dt\right)\right\}$$

s.t.

$$P_2^* = z_1(R - P_6 - \Pi_{C2}) \quad (28)$$

$$P_3^* = z_2(R - P_7 - \Pi_{C3}) \quad (29)$$

$$P_4^* = z_3(R - P_8 - \Pi_{C4}) \quad (30)$$

$$P_5^* = z_4(R - P_{10} - \Pi_{C5}) \quad (31)$$

$$z_1 + z_2 + z_3 + z_4 = 1 \quad (32)$$

4.1.3. Solution procedure

As the objective function and the constraints involve complex and nonlinear functions in both optimization levels, then it needs to solve the bi-level optimization problems using evolutionary algorithms –e. g. genetic algorithm, simulated annealing, etc. (See Konak, et al. [26] and Sinha, et al. [27]). Note that the optimization problems in Esmaili et al. [22] consist of mixed-integer linear functions and hence it can be solved using the backward induction method to gain the solution. Moreover, an approach proposed by Sinha, et al. [27] which allows us to reduce the bi-level problem to a single-level problem using Karush-Kuhn Tucker (KKT) condition. Using this approach, the equation on the lower-level problem of the proposed model becomes the Lagrange equation of the KKT conditions given by:

Lower-level problem: SA-Stackelberg Game.

$$L(P_6,P_7,P_8,P_{10},\boldsymbol{\mu},\boldsymbol{\lambda}) = z_1\left\{P_6 - C_r^A\left(\sum_{i=m+1}^{k}\int_{T_{i-1}}^{T_i} r_{i-1}^O(t)dt\right)\right\} + z_2\left\{P_7 - \sum_{i=m+1}^{k} C_p^A(\delta_A) - C_r^A\left(\sum_{i=m+1}^{k}\int_{T_{i-1}}^{T_i} r_{i-1}^A(t)dt\right)\right\} \quad (33)$$

$$+z_3\left\{P_8 - \sum_{i=m+1}^{k} C_p^A(\delta_A) - E[N_{CM}]\right\} + z_4\left\{P_{10} - E[N_{PM}] - C_r^A\left(\sum_{i=1}^{m}\int_{T_{i-1}}^{T_i} r_{i-1}^A(t)dt\right)\right\} + \boldsymbol{\mu}^T(z_1+z_2+z_3+z_4-1)$$

$$+ \lambda^T(P_2 - z_1(R - P_6 - \Pi_{C2}) + P_3 - z_2(R - P_7 - \Pi_{C3}) + P_4 - z_3(R - P_8 - \Pi_{C4}) + P_5 - z_4(R - P_{10} - \Pi_{C5})$$

$$\nabla_P L(P_6,P_7,P_8,P_{10},\boldsymbol{\mu},\boldsymbol{\lambda}) = 0$$

Equality constraints.

$$\nabla_\mu L(P_6,P_7,P_8,P_{10},\boldsymbol{\mu},\boldsymbol{\lambda}) = 0 \quad (34)$$

Inequality constraints (complementary slackness condition).

$$\lambda_j \geqslant 0 \, for \, j = 1,\ldots,4; \lambda_j z_j \Pi_j = 0$$

Then, to construct a single-level optimization, the conditions of KKT above will be considered as additional constraints for the objective function of the manufacturer in the upper-level problem. After being transformed into a single-level optimization, a genetic algorithm (GA) will be used to carry out the search process for the best solution. GA is chosen as a method for finding solutions to bilevel problems that are categorized as an NP-hard (see Ben-Ayed and Blair [36]), where the objective function considered is non-linear and contains 0–1 variables as in Shuang et al [37]. In addition, GA provides better global solution search capabilities than other traditional methods (see Hejazi et al. [38], and Calvete et al. [39]). With GA, an optimization method and search technique are based on genetic and evolutionary principles [32]. Initially, a population consisting of individuals is generated which is a set of decision variables to be optimized, then the initial fitness value is calculated. After that, evolutionary rules consisting of mating, crossover, and mutation are applied to the appropriate individuals, to create a new set of individuals that provide better fitness values. GA allows a population of many individuals to develop under defined selection rules towards a state that maximizes the "fitness" function. In short, a GA attempts to "evolve" to an optimal solution [40]. In this paper, the proposed GA procedures involve five steps given in Fig. 3, and the five-step GA is described as follows.

Step 0 (Initialization)

The population size chosen is 50 as suggested in De Jongs [28], Grefenstette [29], and Eshelman et al. [30]. Each decision variable will be represented by binary encodings with a length of 30 bits (Goldberg et al. [31]). As the decision variable in the lower-level model becomes a constraint for the upper-level

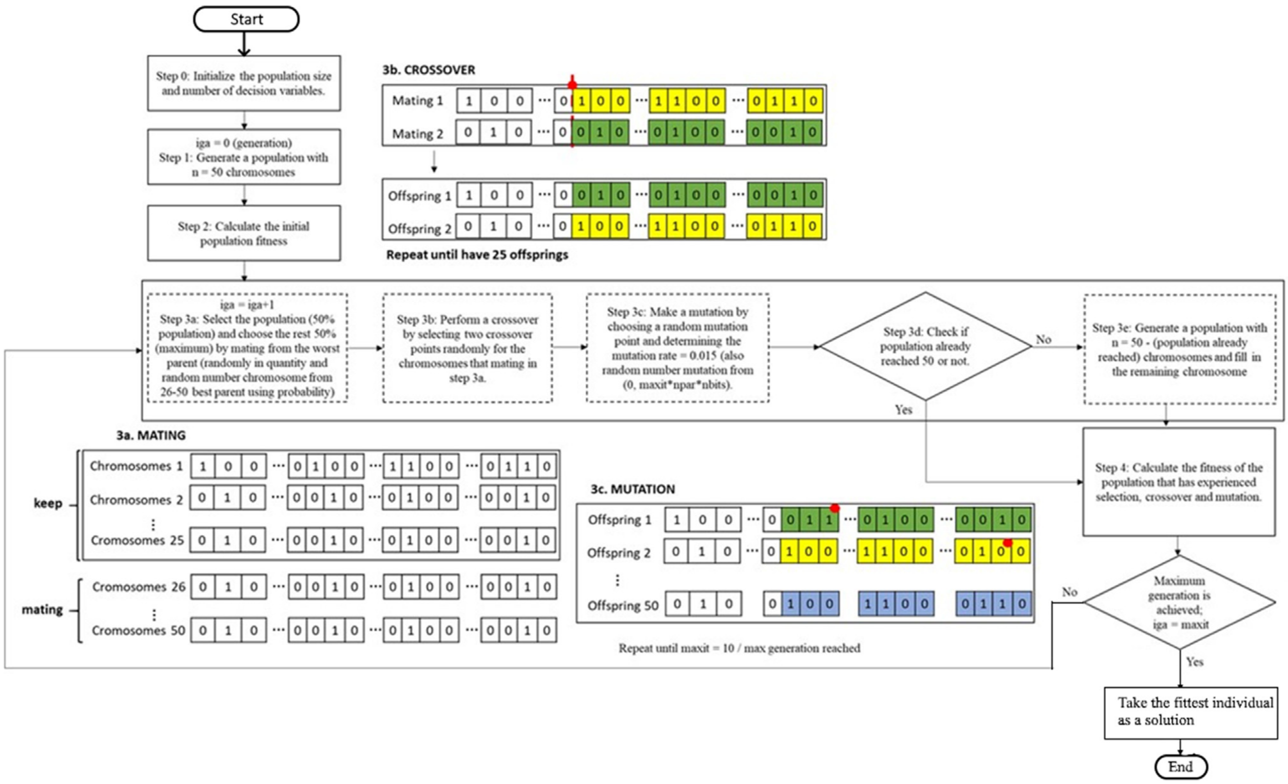


Fig. 3 Flowchart of the genetic algorithm proposed for solving the bi-level optimization.

model, then additional decision variables are needed –i.e., $\boldsymbol{\mu}$ and$\boldsymbol{\lambda}$. So, the total number of variables is 16. Hence, each individual (chromosome) is represented by a 480 (30 × 16) long bit string.

Step 1 (Generating initial population)

The initial population is generated randomly (binary). The population formed is a matrix size (50x480) i.e. the population size (50) multiplied by the total bits of the chromosome (16x30). The population is represented in a matrix of size 50 × 480 which is the population size (50) multiplied by the total bits of the chromosome (480).

Step 2 (Calculating the initial fitness population)

Since the variable values are represented in binary, there must be a way of converting continuous values into binary, and vice versa. After the binary population is formed in Step 1, a conversion is made to convert the value of bits into a real number, and this will be used to calculate the fitness value. Encoding and decoding formula of the nth variable,p_n, from Haupt and Haupt [32] is used in this paper as follows:

For encoding, the following formulas are used to convert the value of a real number into bits,

$$p_{norm} = (p_n - p_{lo})/(p_{hi} - p_{lo}) \tag{36}$$

$$gene[m] = round\left\{ p_{norm} - 2^{-m} - \sum_{p=1}^{m-1} gene[p]2^{-p} \right\} \tag{37}$$

and then for decoding, these quantized functions are used to convert the value of bits into a real number.

$$p_{quant} = \sum_{m=1}^{N_{gene}} gene[m]2^{-m} + 2^{-(M+1)} \tag{38}$$

$$q_n = p_{quant}(p_{hi} - p_{lo}) + p_{lo} \tag{39}$$

where

p_{norm} = normalized variable, $0 \leqslant p_{norm} \leqslant 1$;$p_{lo}$ = smallest *variable value* (7500);p_{hi} = highest variable value (10000);-$gene[m]$ = binary version of p_n;$round\{.\}$ = round to nearest *integer*;p_{quant} = quantized version of p_{norm};N_{gene} = number *bits in a gene*; andq_n = quantized version of p_n. Whenever the fitness function is evaluated, the chromosome must first be decoded using (38). The fitness function used here is given in (21) with constrains (22–26), (27–32), and (33–35). The chromosome that maximizes the fitness function is considered as the best individual.

Step 3a (Mating)

After decoding and calculating the fitness of the population, determine the next population by choosing half of the population that has the best value (i.e. maximum value).

Step 3b (Crossovers)

Half population i.e. 25 chromosomes with the maximum value of the objective function is maintained. Then, form the 26 new chromosomes (so that the total chromosomes become 51). These chromosomes initially come from each chromosome with the lowest objective function value i.e. chromosomes with objective function values from 26 to 50, and the probability values for the 26 sequence chromosomes are set in decreasing order. The goal is to maintain individuals with high fitness values hoping that the crossover results from parents with low fitness values will give offspring with high fitness values.

The selection of each of the two chromosomes is carried out to further determine the crossover point between the two chromosomes of this pair. The first are 13 chromosomes (i^{th} chromosomes), then the second, 13 chromosomes (jth chromosomes) are also selected. Determine the crossover points for the 13 chromosomes using the formula $\mathrm{rand}(1, M) \times (totalbits - 1)$.

Step 3c (Mutation)

The mutation process is done by selecting a random mutation point such as selecting the step 3b crossover point. Previously, determine the number of mutations that will be done by the formula$(popsize - 1) \times$ number bits $\times$ mutation rate. The mutation rate is set at 0.015. Next, generate data to select the column and row addresses (addresses of random variables and generations that state bits) that will be mutated.

Step 4

Calculate the fitness of the population as a result of Step 3. Repeat Step 3 up to the desired maximum value (e.g., 10,000) or until the generation runs out.

4.2. Two-Partite decision problem

In this, the OEM (or the SA) is considered the only maintenance service provider who sells the service directly to the customer. Hence, the OEM (or the SA) is the leader, and the customer is the follower, and the decision problems need to be modelled are (1) the OEM and the consumer for Option 1 and (2) the SA and the consumer for Option 5.

Case (1). *OEM-Consumer Decision Problem.*

Here, the OEM is a monopolist service provider selling the service (Option 1) directly to the customer. Again, Stackelberg game is appropriate to be used in modeling the decision problems of the two parties (i.e., the OEM and the consumer), and this results in a sequential structure described as follows.

The OEM will first take a decision, and then after observing the optimal decision of the OEM, the consumer will take a decision. Using the backward induction method, the optimal solution for this two-stage Stackelberg game is as follows (Murthy and Jack [1]).

Stage 2: The Consumer's decision.

Given the action P_1 previously chosen by the OEM, the consumer will find the best option (take Option 1 or not) that maximizes its profit. The solution to this problem will give the maximum value of the following equation.

The consumer's profit is given by.

$$\Pi_{C1} = R - P_1; R = (r)(L - E[D(t)])$$

$$Max\Pi_{C1} = R - P_1 \tag{40}$$

The consumer's decision to select Option 1 is dependent on Π_{C1} – i.e., (a) does select if $\Pi_{C1} > 0$ and (b) does not select if $\Pi_{C1} < 0$. The consumer is indifferent between decisions (a) and (b) when $\Pi_{C1} = 0$. Hence, the customer will select Option 1 only if $\Pi_{C1} = R - P_1 \geq 0$ or $P_1 \leq R$, and will be indifferent if $P_1 = R$.

Stage 1: The OEM's decision.

The OEM's profit is given by.

$$\rho_{M1}(P_1) = P_1 - E[N_{PM}] - E[N_{CM}]$$

$$Max\rho_{M1}(P_1) = P_1 - E[N_{PM}] - E[N_{CM}] \tag{41}$$

The OEM will sell the maintenance service if its profit, $\rho_{M1}(P_1)$ is positive, and hence P_1 should be greater than the total maintenance cost (consisting of PM and CM costs) or $P_1 > \mathrm{E}[N_{\mathrm{PM}}] + \mathrm{E}[N_{\mathrm{CM}}]$.

As from the consumer's decision that $P_1 \leq R$, then the feasible values of P_1 are given by.

$\mathrm{E}[N_{\mathrm{PM}}] + \mathrm{E}[N_{\mathrm{CM}}] < P_1 \leq R$ which fulfills (40) and (41). The OEM will earn a maximum profit if P_1 approaches R. Note that the consumer's profit is zero if $P_1 = R$ and hence the consumer will be indifferent. One can define P_1 so that the total profit of both parties is shared equally, and hence we have.

$$P_1 - E[N_{PM}] - E[N_{CM}] = R - P_1$$

$$P_1 = \frac{\{R + E[N_{PM}] + E[N_{CM}]\}}{2}$$

This will lead to a win–win solution which is well known as the solution of Nash game formulation for this decision problem (Murthy and Jack [1]). In general, solution using the Stackelberg game formulation will favor a leader (i.e., the OEM). Thus, the OEM will determine the optimal price such that its profit is greater than the consumer's profit.

$$P_1 - E[N_{PM}] - E[N_{CM}] > R - P_1$$

$$P_1 > \frac{\{R + E[N_{PM}] + E[N_{CM}]\}}{2}$$

As a result, the feasible values are given by $\{R + E[N_{PM}] + E[N_{CM}]\}/2 < P_1 < R$.

Case (2). *SA-Consumer Decision Problem.*

The decision situation is similar as in Case 1, but in this case, the SA as a monopolist service provider. Following the backward induction method used in Case (1), the optimal solution for this two-stage Stackelberg game for this case is as follows.

Stage 2: The Consumer's decision.

Given the value of P_9 previously selected by the SA, the consumer will choose the best option (take Option 5 or not) that maximizes its profit. The solution to this problem will give the maximum value of the following equation.

The consumer's profit:

$$\Pi_{C5} = R - P_9; R = (r)(L - E[D(t)])$$

$$Max\Pi_{C5} = R - P_9 \tag{42}$$

The consumer's decision to select Option 5 is dependent on Π_{C5} – i.e., (a) does select if $\Pi_{C5} > 0$ and (b) does not select if $\Pi_{C5} < 0$. The consumer is indifferent between decisions (a) and (b) when $\Pi_{C5} = 0$. Hence, the customer will select Option 5 only if $\Pi_{C5} = R - P_9 \geq 0$ or $P_9 \leq R$, and will be indifferent if $P_9 = R$.

Stage 1: The SA's decision.

The SA's profit:

$$\rho_{A4}(P_9) = P_9 - E[N_{PM}] - E[N_{CM}]$$

$$Max\rho_{A4}(P_9) = P_9 - E[N_{PM}] - E[N_{CM}] \tag{43}$$

The SA will decide to sell the maintenance service if its profit is positive. Hence, the feasible values for P_9 are given by $P_9 > E[N_{PM}] + E[N_{CM}]$. From the consumer's decision we have $P_9 \leq R$, then the feasible values of P_9 given by $E[N_{PM}] + E[N_{CM}] < P_9 \leq R$ which fulfills (42) and (43). The SA's profit reaches the maximum value when P_9 approaches R. This in turn make the consumer's profit is equal to zero as $P_9 = R$ and hence he will be indifferent. Following the similar approach as in Case 1, the Nash game solution of P_9 is given by.

$$P_9 - E[N_{PM}] - E[N_{CM}] = R - P_9$$

$$P_9 = \frac{\{R + E[N_{PM}] + E[N_{CM}]\}}{2}$$

and the Stackelberg game solution is given by.

$$P_9 - E[N_{PM}] - E[N_{CM}] > R - P_9$$

$$P_9 > \frac{\{R + E[N_{PM}] + E[N_{CM}]\}}{2}$$

As a result, the feasible values for P_9 are given by $\{R + E[N_{PM}] + E[N_{CM}]\}/2 < P_9 < R$.

4.3. Numerical example and discussion

The time to the first failure is assumed to follow the Weibull distribution with $F(t) = 1 - \exp(-t/\alpha)^\beta$. Failure rate function is given by $r(t) = \beta(t^{\beta-1}/\alpha^\beta)$. Two sets of PM effect parameters (1 and 2) and three sets of PM cost parameters (A, B, and C) will be used and the values of the parameters are given in Tables 2 and 3. Hence, six sets of parameters will be considered. The difference of sets 1 and 2 is in the value representing the quality of PM action by the SA, ψ, 1.05 and 1.15 respectively (See Table 2). The other parameters are kept the same in sets 1 and 2 – i.e., $\delta_O = 0.00166$; $\delta_A = 0.001494$; $W = 360$; $\tau = 360$; $\mu = 1.2$; and $\beta = 2.05$, r (the revenue generated from the equipment when it operates) = 45.3 per unit time. Meanwhile, the difference between sets A, B, and C is in the percentage of CM cost of SA, which is lower than the CM cost of OEM. The PM costs formula are $C_0^O = 1.50 \times C_r^O$ and $C_1^O = 0.50 \times C_r^O$, where a fixed cost (or an overhead cost) for SA, in general, is less than a fixed cost for OEM. For Set 1 and 2, the value of ψ representing the quality of PM action by the SA are ψ = 1.05 and 1.15 (or > 1.0) meaning that the effect of PM done by SA is less effective compared to that of the OEM as the OEM has more know how in performing the PM.

Table 2 Parameter values associated with the PM effect.

In Party	Maintenance Parameter	Set	
		1	2
OEM	ψ	1.00	1.00
	δ_O	0.00166	0.00166
SA	ψ	**1.05**	**1.15**
	δ_A	0.001494	0.001494

Table 3 Parameter values for fixed and variable costs of a PM.

Party	Cost Parameter	Set		
		A	B	C
OEM	C_r^O	1.00	1.00	1.00
	C_0^O	1.50	1.50	1.50
	C_1^O	0.50	0.50	0.50
SA	C_r^A	0.40	0.60	**0.80**
	C_0^A	0.60	0.90	**1.20**
	C_1^A	0.20	0.30	**0.40**

First, it needs to show that the GA algorithm developed is able to produce a convergence solution, and then the optimal solution obtained using the developed GA. Using parameters Set 2 and Set B (called Set 2B), the values of the objective function obtained are plotted and there are shown in Fig. 4. The findings from Fig. 4 are described as follows.

- It can be seen from Fig. 4 that each plot of the value of objective function leads to a convergent point (it reaches a stable value).
- The larger the population size, the faster the model solution goes to the convergence point. It can be seen that the objective function value reaches the point of convergence at the 20th, 55 th, 69 th, and 70 th generation for the populations size of 30, 40, 50, and 60, respectively. Meaning that the larger of the population size the faster the solution will get a convergent point.
- The GA algorithm developed is able to produce the convergence solution, and hence it can be used to solve other sets of parameter values.

Tables 4A-4C show results for MSC Options 2, 3, 4, and 6 obtained from GA using data Set 1. The results using data Set 2 are shown in Tables 5A-5C. The profit value typed in bold font is the best profit for the OEM, and the option associated with the best profit is defined as the best option for the consumer -e.g., as shown in Table 4A that the best profit for the OEM is **8812.89** which is generated by selling Option **4**, and this option is defined as the best MSC option. The results for the three - partite and the two - partite decision problems are presented sequentially as follows.

The findings from a three - partite decision problem:

- For Set 1A (the PM quality done by the SA is slightly lower than the quality PM of the OEM), the best MSC option for the consumer's equipment (e.g., dump trucks operated in a mining company) is Option 4 (i.e., the OEM performs PM in $(0, W)$ and the SA carries out PM in (W, L), and CM in $(0, L)$). The OEM and the SA will earn profit of 8812.89 and 8720.71 from selling Option 4, respectively. This is due to the use of Stackelberg game (SG) which gives an advantage to the OEM (as a leader) for deciding the optimal price prior to the decision of SA. The consumer must pay 17543.74 for this MSC option over L, which is the sum of $P_4 = 8815.39$ and $P_8 = 8728.35$, and will earn a profit of 14711.35.

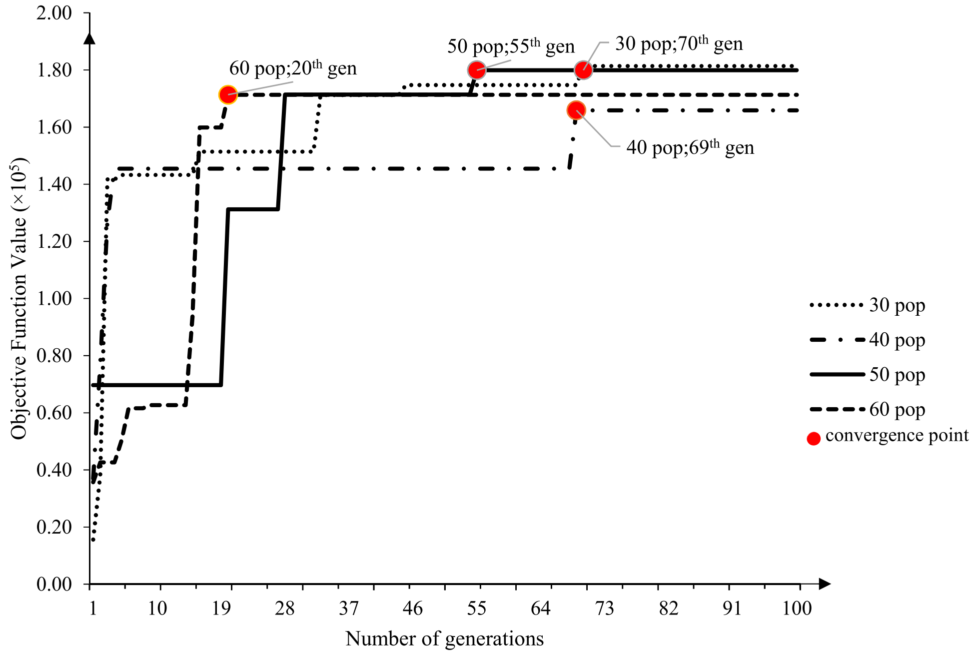


Fig. 4 Convergence graph for option 1 OEM (100 generations).

Table 4A GA results (contract prices and profits) for Set 1A.

MSC Options	Party Involved	Contract Price	Profit	Consumer's Profit
2	OEM	P_2 = 8737.29	8730.88	14650.42
	SA	P_6 = 8867.39	8863.65	
3	OEM	P_3 = 8768.28	8762.88	14851.77
	SA	P_7 = 8635.04	8630.31	
4	OEM	P_4 = 8815.39	**8812.89**	14711.35
	SA	P_8 = 8728.35	8720.71	
6	OEM	P_5 = 8764.41	8751.22	14054.46
	SA	P_{10} = 8723.25	8720.62	

Table 4B GA results (contract prices and profits) for Set 1B.

MSC Options	Party Involved	Contract Price	Profit	Consumer's Profit
2	OEM	P_2 = 8588.02	8581.63	14840.93
	SA	P_6 = 8724.56	8718.96	
3	OEM	P_3 = 8711.33	8705.94	14687.74
	SA	P_7 = 8754.44	8747.34	
4	OEM	P_4 = 8814.60	**8812.11**	14668.41
	SA	P_8 = 8670.50	8660.49	
6	OEM	P_5 = 8693.93	8680.74	13969.54
	SA	P_{10} = 8878.65	8874.71	

- For Set 1B and 1C, where the PM and CM costs of the SA increase (but the costs are still lower than the costs charged by the OEM), the best option is still Option 4. In other words, the increase in the maintenance costs of the SA will not influence the best decision as long as the costs are still low and the quality of PM and CM remains comparable to that of the OEM. All prices and profits of the SA are relatively stable (as the relative different is less than 1%) with the increase in the SA's maintenance costs. As under the three stage SG, in the upper-level problem, OEM takes a decision before the SA does, and in the lower-level problem, the SA decides the price before the consumer chooses the best decision, then the OEM can enforce his decision on the other parties in determining the optimal price maximizing his profit although the cost structure of the SA is getting larger. Hence, the OEM's profit is maintained at least 8812

Table 4C GA results (contract prices and profits) for Set 1C.

MSC Options	Party Involved	Contract Price	Profit	Consumer's Profit
2	OEM	$P_2 = 8733.83$	8727.43	14681.40
	SA	$P_6 = 8636.70$	8629.23	
3	OEM	$P_3 = 8674.29$	8668.89	14594.46
	SA	$P_7 = 8783.19$	8773.72	
4	OEM	$P_4 = 8870.13$	**8867.64**	14440.53
	SA	$P_8 = 8741.26$	8728.89	
6	OEM	$P_5 = 8763.50$	8750.32	14098.02
	SA	$P_{10} = 8680.59$	8675.34	

Table 5A GA results (contract prices and profits) for Set 2A.

MSC Options	Party Involved	Contract Price	Profit	Consumer's Profit
2	OEM	$P_2 = 8760.57$	8754.17	14690.31
	SA	$P_6 = 8784.03$	8779.92	
3	OEM	$P_3 = 8704.27$	8698.86	14717.00
	SA	$P_7 = 8813.65$	8808.54	
4	OEM	$P_4 = 8789.93$	**8787.43**	14692.65
	SA	$P_8 = 8752.33$	8744.32	
6	OEM	$P_5 = 8735.28$	8719.29	13983.13
	SA	$P_{10} = 8823.70$	8820.96	

Table 5B GA results (contract prices and profits) for Set 2B.

MSC Options	Party Involved	Contract Price	Profit	Consumer's Profit
2	OEM	$P_2 = 8757.30$	8750.90	14682.68
	SA	$P_6 = 8683.26$	8677.10	
3	OEM	$P_3 = 8800.83$	**8795.43**	14436.50
	SA	$P_7 = 8885.92$	8878.26	
4	OEM	$P_4 = 8708.24$	8705.74	14688.53
	SA	$P_8 = 8726.48$	8715.91	
6	OEM	$P_5 = 8669.58$	8653.59	14083.86
	SA	$P_{10} = 8788.68$	8784.56	

Table 5C GA results (contract prices and profits) for Set 2C.

MSC Options	Party Involved	Contract Price	Profit	Consumer's Profit
2	OEM	$P_2 = 8668.56$	8662.16	14610.69
	SA	$P_6 = 8732.34$	8724.12	
3	OEM	$P_3 = 8774.26$	**8768.86**	14483.92
	SA	$P_7 = 8753.40$	8743.19	
4	OEM	$P_4 = 8698.57$	8696.07	14587.28
	SA	$P_8 = 8725.73$	8712.61	
6	OEM	$P_5 = 8780.63$	8764.64	14100.64
	SA	$P_{10} = 8660.85$	8655.36	

for sets 1A-1C, and this in turn increase the total contract price in one side, and decrease the profit of the consumer, in the other side as the values of SA's PM and CM costs for Set 1C are double from the corresponding values for Set 1A (See Table 6A).

- For Set 2 where the PM quality of the SA is much lower, the best decision is Option 4 for Set 2A (when maintenance costs are lower), and then change to Option 3 (for sets 2B and 2C) in which the OEM provides CM as well as PM in$(0, W)$. In other words, both the OEM and the SA provide a package of PM and CM - i.e., the OEM performs a package of PM and CM in $(0, W)$ whilst the SA in(W, L). This is due to the PM quality by SA ($\psi = 1.15$) is lower than the PM quality of the OEM ($\psi = 1.00$) but the PM and CM costs charged are slightly lower than the maintenance costs of the OEM. This in turn will make the option in which the OEM performs both PM and CM in $(0, W)$ is the best choice (See Table 6B).

Table 6A Best option for Set 1.

Data	Best Options	Party Involved	Contract Price	Total Contract Price	Profit	Consumer's Profit
Set 1A	Option 4	OEM	P_4 = 8815.39	17543.74	**8812.89**	14711.35
		SA	P_8 = 8728.35		8720.71	
Set 1B	Option 4	OEM	P_4 = 8814.60	17485.10	**8812.11**	14668.41
		SA	P_8 = 8670.50		8660.49	
Set 1C	Option 4	OEM	P_4 = 8870.13	17611.39	**8867.64**	14440.53
		SA	P_8 = 8741.26		8728.89	

Table 6B Best option for Set 2.

Set Data	Best Options	Party Involved	Contract Price	Total Contract Price	Profit	Consumer's Profit
Set 2A	Option 4	OEM	P_4 = 8789.93	17542.26	**8787.43**	14692.65
		SA	P_8 = 8752.33		8744.32	
Set 2B	Option 3	OEM	P_3 = 8800.83	17686.75	**8795.43**	14436.50
		SA	P_7 = 8885.92		8878.26	
Set 2C	Option 3	OEM	P_3 = 8774.26	17527.66	**8768.86**	14483.92
		SA	P_7 = 8753.40		8743.19	

- This finding shows that if the SA has a high quality of maintenance, then the SA will get more portion in providing the maintenance service as long as the maintenance costs is kept lower compared to the costs of the OEM.
- If the SA's maintenance quality decreases (but it still a good quality) and the maintenance costs are kept low, then the portion of the MSC remains the same. But if the quality of maintenance decreases and the maintenance costs get larger (i.e., 60 to 80% of the costs of the OEM), then the portion of the MSC will get smaller. This is due to the moderate quality of maintenance services charged by the SA is relatively high.
- As in the results for sets 1A-1C, the prices of MSC options offered by SA and the associated profits for sets 2A-2C are relatively stable (as the changes (measured in a relative difference) are very small i.e., less than 1%) with the increase in the SA's maintenance costs.

The findings from a two-partite decision problem:

- For Options 1, the feasible values of P_1 fall within 16001.90 $< P_1 <$ 31988.80 for all sets of data used as the maintenance cost structure of the OEM is not influenced by the change in parameter values under different sets considered. The OEM will decide P_1 so that to obtain the maximum profit, and hence
- $P_1 = 31988.80$ which is very high and will not be a competitive price when there is a service agent (who is able to provide a good quality of the maintenance services for the equipment with a lower price) in the market. Note that the highest price of best MSC option (which combines an option of the OEM and an option of the SA) is 17611.39 for sets 1A-1C and 17686.75 for sets 2A-2C, which is very much lower than the price of Option 1.
- For Options 5, the feasible values of P_9 fall within 15891.80 $< P_9 <$ 31777.20 for all sets of data considered. The lower bound and upper bound is slightly lower than those of Option 1 as the revenue received by the consumer is affected by the maintenance cost structure and the PM and CM quality of the SA. Under a monopolistic market, the SA may select $P_9 = 31777.20$ as the SA will get the maximum profit. But, again, this price will not be competitive under a more competitive market as the highest price of the best MSC option is around 17,600 for sets 1A-1C and for sets 2A-2C. The lower price of the best MSC option in the three-partite decision problem is a result of the combination of two best practices – i.e., the best maintenance quality of the OEM and the competitive cost structure of the SA.

5. Conclusion

In this research, six maintenance service contract options (involving the OEM, the SA, and the consumer) for repairable products have been studied. The study of these MSCs is done from three different perspectives — the OEM, the SA, and the customer, and the goal for this study is to determine the optimal price of each option (for the OEM and/or the SA) and to select best option (for the consumer), which maximize profits for the three parties involved. Findings obtained from the numerical results are as follows.

- For the case where the SA has a good quality of maintenance, then the best MSC option for the consumer's equipment is Option 4 where the SA will get more portion in providing the maintenance service over L provided that the maintenance costs are kept lower compared to the costs of the OEM.
- If the SA's maintenance quality decreases and the maintenance costs are still low, then the portion of the MSC remains the same. But if the quality of maintenance is moderate and the maintenance costs are relatively high, then the portion of the MSC will get smaller.
- The price of a MSC option is favor the OEM or the SA when the market is monopolistic - i.e., when the OEM or the SA is the only maintenance service provider in the market.

- When more alternatives of MSC options (involving the OEM and the SA) are available for the consumer to be chosen or the market is more competitive, then the price of the maintenance services tends to be lower, and this is favoring the consumer.

In this paper, the warranty policy considered is a one-dimensional warranty, then one can extend to a two-dimensional case (i.e., both warranty and MSC are characterized by two parameters – i.e., age and usage). Other interesting topics are (i) to examine a warranty and MSC for a fleet of equipment and (ii) to use a particle swarm optimization (PSO) to solve the bi-level decision models for the MSC options proposed as the PSO is claimed as the efficient algorithm for bilevel optimization problems (Kuo and Huang [45], Jiang et al. [46]) since GA can produce better global solutions but it is not an efficient method (Hejazi et al. [38]) (e.g., time required to run one set of data for this study is 23 h). The research of these topics is on the way.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

We thank the anonymous referees for their critical and constructive comments that helped to significantly improve the content and the presentation of the manuscript. This research is funded by P2MI (Research, community services, and innovation), Faculty of Mathematics and Natural Sciences, ITB with contract number 62/IT1.C02/SK-TA/2021.

References

[1] D.N.P. Murthy, N. Jack, Extended Warranties, Maintenance Service and Lease Contracts: Modeling and Analysis for Decision-Making, Springer-Verlag, London, 2014.
[2] K. Laksana, J.C. Hartman, Planning product design refreshes with service contract and competition considerations, Int. J. Prod. Econ. 126 (2) (2010) 189–203.
[3] Z. Lian, J. Wu, Design and analysis of a maintenance service contract, IET Conference Publications 2013 (644) (2013) 110–115.
[4] B.P. Iskandar, U.S. Pasaribu, H. Husniah, Maintenance service contract for equipment sold with two-dimensional warranties, J. Qual. Technol. Quality Manage. 11 (3) (2014) 321–333.
[5] H. Husniah, U.S. Pasaribu, B.P. Iskandar, Service contract management with availability improvement and cost reduction, ARPN Journal of Engineering and Applied Sciences 10 (1) (2015) 146–151.
[6] J.M.M. de Santana, R.L.V. Santiago, I.D. Lins, M. das Chagas Moura, and E.L. Droguett,, A principal-agent approach for designing maintenance service contracts, in: Proceedings of the 29th European Safety and Reliability Conference ESREL, 2019, 2019,, pp. 460–466.
[7] K. Rinsaka, H. Sandoh, A stochastic model on an additional warranty service contract, Computers and Mathematics with Application 51 (2006) 179–188.
[8] D. Zhang, Q. Xu, H. Song, A warranty service contract among manufacturer and agent, in: 2015 8th International Symposium on Computational Intelligence and Design (ISCID), 2015, pp. 568–571.
[9] P.S. Desai, V. Padmanabhan, Durable good extended warranty and channel coordination, Rev. Mark. Sci. 2 (2) (2004) 1–23.
[10] K. Li, S. Mallik, S. Chahajed, Design of extended warranties in supply chains under additive demand, Prod. Oper. Manag. 21 (2012) 730–746.
[11] H. Kurata, S.H. Nam, After-sales service competition in a supply chain: optimization of customer satisfaction level or profit or both?, Int J. Prod. Econ. 127 (2010) 136–146.
[12] B. Jiang, X. Zhang, How does a retailer's service plan affect a manufacturer warranty?, Manage Sci. 57 (2011) 727–740.
[13] S.H. Heese, Retail strategies for extended warranty sales and the impact of manufacturer base warranties, Decis. Sci. 43 (2012) 341–367.
[14] S. Kameshwaran, N. Viswanadham, V. Desai, Bundling and pricing of product with aftersale services with product, Int. J. Oper. Res. 6 (2009) 92–109.
[15] N. Jack, D.N.P. Murthy, A flexible extended warranty and related optimal strategies, Journal of the Operational Research Society. 58 (2007) 1612–1620.
[16] Y. Lam, P.K.W. Lam, An extended warranty policy with options open to consumers, Eur. J. Oper. Res. 131 (2001) 514–529.
[17] J.C. Hartman, K. Laksana, Designing and pricing menus of extended warranty contracts, Nav. Res. Logistics. 56 (2009) 199–214.
[18] H. Tarakci, K. Tang, H. Moskowitz, R. Plante, Incentive maintenance contracts for channel coordination, IIE Trans. 38 (2006) 671–684.
[19] H. Tarakci, K. Tang, H. Moskowitz, R. Plante, Maintenance outsourcing of a multi-process manufacturing system with multiple contractors, IIE Trans. 38 (2006) 67–78.
[20] H. Tarakci, K. Tang, S. Teyarachakul, Learning effects on maintenance outsourcing, Eur. J. Oper. Res. 192 (2009) 138–150.
[21] N.S. Gamchi, M. Esmaili, dan M.S. Monfared, Three-level service contract between manufacturer, agent and customer (game theory approach), Proceeding of the 2012 International Conference of Industrial Engineering and Operation Management July 3-6 (2012) Turkey.
[22] M. Esmaili, N.S. Gamchi, E. Asgharizadeh, Three-level warranty service contract among manufacturer, agent and customer: A game-theoretical approach, European Journal of Operational Research (2014) 177-186.
[23] R.E. Barlow, L.C. Hunter, Reliability analysis of a one-unit system, Operation Research 8 (1) (1961) 200–208.
[24] R. Jiang, D.N.P. Murthy, Maintenance: decision models for management, Science Press, Beijing China, 2008.
[25] B.P. Iskandar, H. Husniah, Optimal preventive maintenance for a two dimensional lease contract, Comput. Ind. Eng. 113 (2017) 693–703.
[26] A. Konak, S. Kulturel-Konak, L.V. Snyder, A game-theoretic genetic algorithm for the reliable server assignment problem under attacks, Comput. Ind. Eng. 85 (2015) 73–85.
[27] A. Sinha, P. Malo, K. Deb, Evolutionary algorithm for bi-level optimization using approximations of the lower level optimal solution mapping, Eur. J. Oper. Res. 27 (2) (2017) 395–411.
[28] K.A. De Jong, An analysis of the behavior of a class of genetic adaptive systems, University of Michigan, Ann Arbor, 1975, Ph. D. thesis.
[29] J.J. Grefenstette, Genetic algorithms and their applications, Proceedings of the Second International Conference on Genetic Algorithms, 1987.
[30] L.J. Eshelman, R.A. Caruana, J.D. Schaffer, Biases in the landscape, Proceedings of the Third International Conference on Genetic Algorithms, Morgan Kaufmann, 1989.
[31] D.E. Goldberg, K. Deb, B. Korb, Messy genetic algorithms revisited: Studies in mixed size and scale, Complex Systems 4 (1990) 415–444.
[32] R.L. Haupt, S.E. Haupt, Practical Genetic Algorithms, John Wiley & Sons Inc, Hoboken, New Jersey, 2004.

[33] H. Huang, F. Liu, P. Zhang, To outsource or not to outsource? Warranty service provision strategies considering competition, costs and reliability, International, J. Product. Econ. 242 (2021) 108298.

[34] T.S. Baines, H.W. Lightfoot, S. Evans, A. Neely, R. Greenough, J. Peppard, R. Roy, E. Shehab, A. Braganza, A. Tiwari, J.R. Alcock, J.P. Angus, M. Bastl, A. Cousens, P. Irving, M. Johnson, J. Kingston, H. Lockett, V. Martinez, P. Michele, D. Tranfield, I.M. Walton, H. Wilson, State-of-the-art in product-service systems, Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture 221 (10) (2007) 1543–1552.

[35] M. Allain, The balance of power between producers and retailers; a differentiation model, Recherches économiques de Louvain 68 (2002) 359–370.

[36] O. Ben-Ayed, C.E. Blair, Computational difficulties of bilevel linear programming, Oper. Res. 38 (3) (1990) 556–560.

[37] S. Ma, G. Du, J. Jiao, R. Zhang, Hierarchical game joint optimization for product family-driven modular design, Journal of the Operational Research Society 67 (12) (2016) 1496–1509.

[38] S.R. Hejazi, A. Memariani, G. Jahanshahloo, M.M. Sepehri, Linear bilevel programming solution by genetic algorithm, Comput. Oper. Res. 29 (2002) 1913–1925.

[39] H.I. Calvete, C. Gale, P.M. Mateo, A new approach for solving linear bilevel problems using genetic algorithms, Eur. J. Oper. Res. 188 (1) (2008) 14–28.

[40] D. Goldberg, Genetic Algorithms in Search, Optimization, and Machine Learning, Addison-Wesley Publishing Company, Reading, 1989.

[41] J.F. Bard, Practical Bilevel Optimization: Algorithms and Applications, Springer, Boston, 1998.

[42] S. Dempe, J. Dutta, Is bilevel programming a special case of a mathematical program with complementarity constraints?, Math Program. 131 (2012) 37–48.

[43] A. Sinha, Z. Lu, K. Deb, P. Malo, Bilevel optimization based on iterative approximation of multiple mappings, J. Heuristics 26 (2) (2020) 151–185.

[44] T. Basar, G.J. Olsder, Dynamic Noncooperative Game Theory, Academic Press, London New York, 1982.

[45] R.J. Kuo, C.C. Huang, Application of particle swarm optimization algorithm for solving bi-level linear programming problem, Comput. Math. Appl. 58 (2009) 679–685.

[46] Y. Jiang, X. Li, C. Huang, X. Wu, Application of particle swarm optimization based on CHKS smoothing function for solving nonlinear bilevel programming problem, Appl. Math. Comput. 219 (2013) 4332–4339.